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A PRIZE TO GIVE FOR: AN EXPERIMENT ON PUBLIC GOOD FUNDING MECHANISMS*

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This article investigates fund-raising mechanisms based on a prize as a way to overcome free riding in the private provision of public goods. We focus on an environment characterised by income heterogeneity and incomplete information about income levels. Our analysis compares experimentally the performance of a lottery, an all-pay auction and a benchmark voluntary contribution mechanism. We find that prize-based mechanisms perform better than voluntary contribution in terms of public good provision. Contrary to the theoretical predictions, contributions are significantly higher in the lottery than in the all-pay auction, both overall and by individual income types.

Finding effective fund-raising mechanisms for the private provision of public goods is an important policy issue. Voluntary contributions to public goods are typically well below socially optimal levels, given the incentive to free ride associated with positive externalities; see Ledyard (1995) for a survey. While fund-raising mechanisms based on tax rewards and penalties can be designed to overcome the incentive to free ride, they are not available to fund-raisers in the private sector who cannot enforce sanctions. A number of recent studies have examined, both theoretically and empirically, the performance of incentive-based funding mechanisms for the private provision of public goods, focusing in particular on lotteries (or raffles) and different types of auctions (Morgan, 2000; Morgan and Sefton, 2000; Goeree *et al.*, 2005; Landry *et al.*, 2006; Lange *et al.*, 2007; Orzen, 2008; Schram and Onderstal, 2009).

In this article we investigate with a laboratory experiment the performance of prize-based mechanisms for the private provision of public goods, in a setting characterised by income heterogeneity and incomplete information about income levels. We focus on a voluntary contribution mechanism, used as a benchmark, and two incentive-based mechanisms where a single prize is awarded: a lottery and an all-pay auction. The experimental literature on incentive-based fund-raising mechanisms has focused on the case of income homogeneity (Morgan and Sefton, 2000; Orzen, 2008; Schram and Onderstal, 2009). However, actual contribution to public goods is generally characterised by heterogeneous incomes which are private information. Although several experimental studies have investigated public good provision when incomes are heterogeneous, this literature has only explored the voluntary contribution mechanism.[1] The performance of incentive-based fund-raising mechanisms when subjects have different incomes remains empirically unexplored.

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[1] Research has examined the effects of income heterogeneity on either total public good provision (Anderson *et al.*, 2008; Chan *et al.*, 1996, 1999; Rapoport and Suleiman, 1993) or contributions by individual income types (Buckley and Croson, 2006).

Morgan (2000) provides a theoretical analysis of lotteries as a way to finance public goods. Players buy tickets of a lottery with a single prize. One ticket is randomly drawn and the holder wins the prize. Public good provision consists of the revenue of the lottery net of the prize. The author considers agents, with heterogeneous preferences and endowments, who have quasi-linear utility functions. Public good provision is shown to be strictly higher than with voluntary contributions. In the solution identified by Morgan (2000), agents with different incomes contribute the same amount in equilibrium. Morgan and Sefton (2000) investigate experimentally the performance of a linear version of Morgan's model, finding that, as predicted, public good provision via a lottery is higher than through voluntary contributions. However, they only consider homogeneous endowments, so that the prediction of a symmetric equilibrium when endowments are heterogeneous cannot be tested.

Orzen (2008) compares in a laboratory experiment the performance of a lottery and different all-pay auctions as fund-raising mechanisms, under the assumptions of homogeneous preferences and endowments. Public good provision generated with the incentive-based mechanisms, net of the cost of the prize, is higher than with voluntary contribution. Interestingly, although theory predicts that the first price all-pay auction raises a higher revenue than the lottery, no significant difference is found between the two treatments. Finally, Schram and Onderstal (2009) present an experimental study that compares a winner-pay auction, an all-pay auction and a lottery in the case of heterogeneous preferences but homogeneous endowments.[2] They find that the all-pay auction performs better than the lottery, as predicted by the theory. In sum, all of these studies focus on the case of homogeneous endowments.

Our analysis is based on a theoretical framework where a prize is used as a means to finance public goods when agents have heterogeneous endowments, which are private information. In this setting, an all-pay auction generates a higher expected total contribution than a lottery with an equal prize. The equilibrium of the lottery is completely symmetric, as in Morgan (2000), with all agents contributing the same amount independently of their endowment. In the all-pay auction, we characterise a quasi-symmetric equilibrium where agents with the same endowment randomise according to the same distribution function but agents with higher endowments contribute more in expectation.

In our experiment, we test three main theoretical predictions. First, incentive-based mechanisms should outperform the voluntary contribution mechanism in terms of net contributions (after taking into account the cost of prizes). Second, the total revenue of the all-pay auction should be higher than that of a lottery with an equal prize. Third, absolute contributions should not depend on income in the lottery, whereas they should rise with income in the all-pay auction. As a consequence, individual contributions should be higher in the lottery than in the all-pay auction at the lower end of the income distribution, while the opposite should hold at the upper end.

[2] It should be observed that there is often a duality between changes in preferences and in the available lotteries. Under reasonable assumptions, heterogeneity of preferences as in Schram and Onderstal (2009) can be equivalent to income heterogeneity. We are grateful to an anonymous referee for pointing this out.

Our findings can be summarised as follows. In all mechanisms, average contributions are generally higher than theoretical predictions. Nevertheless, we observe different trends over time. In the all-pay auction, total contributions converge towards the level predicted by the theory, whereas in the lottery they remain well above the theoretical prediction. The introduction of a prize as an incentive has significant effects on contributions: the lottery and, to a lesser extent, the all-pay auction perform better than the voluntary contribution mechanism in terms of public good provision after accounting for the cost of the prize. Comparing the prize-based mechanisms, contributions are significantly higher in the lottery than in the all-pay auction, contrary to the theoretical predictions. Focusing on the behaviour of individual income types, absolute contributions rise with income in all treatments, although more steeply in the prize-based mechanisms. In terms of relative performance, the lottery does better than the other mechanisms for all income types.

The discrepancies between the theoretical predictions and the experimental results may suggest a number of possible explanations. We propose an interpretation of the results based on the hypothesis that a higher degree of competition tends to crowd out agents' concern for social efficiency. We show that a simple extension of the standard utility function, taking into account the effect of competition on agents' concern for social efficiency, allows us to explain the results in all of our experimental treatments. We also show that this framework can be used to explain some of the results of closely related experiments in the literature.

The article is structured as follows. Section 1 describes the experimental design and procedures. Section 2 discusses the theoretical predictions and the hypotheses to be tested. Section 3 presents the results. Section 4 provides an interpretation of the results. Section 5 concludes with a discusion of the main findings and implications of the analysis.

1. The Experiment

Our experimental design is closely related to those in Morgan and Sefton (2005) and Orzen (2008), while introducing income heterogeneity and incomplete information about the income of other subjects. We compare three treatments in a between-subjects design: a voluntary contribution mechanism (VCM), a lottery (LOT) and an all-pay auction (APA). We run three sessions for each treatment, with 16 subjects participating in each session, for a total of 144 subjects. Each session consists of 20 rounds.

1.1. *Baseline Game*

The baseline game is a standard linear public good game, with the introduction of income heterogeneity and incomplete information about income levels. At the beginning of each session the sixteen subjects are randomly and anonymously assigned an endowment of either 120, 160, 200, or 240 tokens. The subjects are informed that in each round they will receive the same endowment as determined at the beginning of the session.

Incomplete information about incomes is introduced by using a variant of the *strangers* matching procedure used by Andreoni (1988). At the beginning of each round, subjects are randomly and anonymously rematched in groups of four players. Therefore, in each period subjects do not know the identity and the endowment of the other three members of their group. They only know that the endowment of each of the other group members can be either 120, 160, 200, or 240 tokens with equal probabilities.[3]

In each round, every subject has to allocate entirely a given endowment between two accounts. The language used in the instructions does not refer to contributions or public goods but asks subjects to allocate tokens to either an 'individual account' or a 'group account'. Subjects privately benefit from what they allocate to the individual account, whereas contributions to the group account are multiplied by two and shared equally among the four members of a group. In order to avoid decimals, returns from both accounts are multiplied by two and expressed in points. Therefore, a subject receives 2 points for each token he allocates to the individual account, while he receives 1 point for each token allocated by him, or by any other member of his group, to the group account.

1.2. *Treatments*

The three treatments differ in the way prizes (extra points) can be earned by the subjects in each round:

- VCM The experimenter allocates exogenously 120 tokens to the group account, independently of the subjects' choices, thus implying that each member of the group receives 120 extra points.
- LOT A subject receives a lottery ticket for each token he allocates to the group account. At the end of each round the computer randomly selects one ticket among those purchased by the members of the group, and the owner of the selected ticket wins a prize of 240 points. In the case where no tokens are allocated to the group account, the winner of the prize is selected randomly among the four members of the group.
- APA The member of the group who allocates the highest amount to the group account wins a prize of 240 points. In the case of ties between two or more group members, the winner is determined randomly by the computer. In case no tokens are allocated to the group account, the winner is selected randomly among the four members of the group.

Note that the three mechanisms imply the same financial commitment for the fund-raiser: allocating 120 tokens to the group account in VCM is equivalent to paying a prize of 240 points in APA or in LOT.

[3] In each round group matching is determined randomly before the beginning of the experiment in the following way. Four pools of four subjects are formed, each containing the four different income types (120, 160, 200 or 240 tokens). Each of the four groups is formed by randomly drawing one subject from each pool. As a consequence, within every group each member can have an endowment of 120, 160, 200, or 240 tokens with equal probability. Having formed the four groups for each round in this way, the same sequence of group matchings for the twenty rounds is used in each session of all three treatments.

1.3. *Procedures*

In each session, the subjects were randomly assigned to a computer terminal at their arrival. To ensure public knowledge, instructions were distributed and read aloud (see Appendix A for the instructions). Moreover, to ensure understanding of the experimental design, sample questions were distributed and the answers were privately checked and, if necessary, individually explained to the subjects.

At the end of each round, the subjects were informed about their payoffs from the group account, the individual account and the prize (or bonus in VCM). At the end of the last round, subjects were informed of their total payoff for the twenty rounds expressed in points and in euro. They were asked to answer a short questionnaire about socio-demographic background, and were then paid in private using an exchange rate of 1000 points per euro.

Subjects earned 12.25 euro on average for sessions lasting about 50 minutes, including the time for instructions. Participants were undergraduate students of Economics recruited by e-mail using a list of voluntary potential candidates. The experiment took place in May 2006 in the Experimental Economics Laboratory of the University of Milan Bicocca. The experiment was computerised using the z-Tree software (Fischbacher, 2007).

2. Theoretical Predictions and Hypotheses

We assume that players are risk-neutral and choose their contributions in order to maximise their expected utility. In VCM, the payoff in points for player i with endowment ω_i is given by

$$2(\omega_i - g_i) + (120 + g_i + G_{-i})$$

where g_i is the player's contribution to the group account and G_{-i} is the sum of the contributions of all the other players in his group. Since the marginal cost of contributing to the public good exceeds the marginal return of investing, the unique Nash equilibrium is for all players to contribute zero tokens, although it is socially optimal to contribute the whole endowment.[4]

In LOT, player i wins the prize with a probability equal to the ratio between his contribution and the total contribution in his group $g_i/(g_i+G_{-i})$. The expected payoff in points for player i is given by

$$2(\omega_i - g_i) + \frac{g_i}{g_i + G_{-i}}240 + (g_i + G_{-i})$$

As we are considering a game where constraints are non-binding for all agents, the lottery has a unique pure strategy equilibrium where each player contributes the same amount, independently of his endowment, as in Morgan (2000).

PROPOSITION 1. *LOT has a unique pure strategy equilibrium where each agent contributes* $g^{LOT} = 45$ *tokens. Total group contribution is* $G^{LOT} = 180$ *tokens.*

[4] Note that, while expected payoffs are expressed in points, theoretical predictions are expressed in tokens.

Proof. See Appendix B.

Note that total contribution is higher than the cost of the prize (120 tokens): the lottery provides positive net revenues.

In APA, a player wins the prize if and only if his contribution is higher than the contributions of all other agents in his group, while ties are randomly broken. The expected payoff in points for player i is given by

$$2(\omega_i - g_i) + \mathrm{E}(240, g_i, \mathbf{g}_{-i}) + (g_i + G_{-i}) \quad (1)$$

where the second term is the expected prize and $\mathbf{g}_{-i}$ is the vector of contributions of the other group members. Expression (1) can be written as

$$2\omega_i + G_{-i} + \mathrm{E}(240, g_i, \mathbf{g}_{-i}) - g_i \quad (2)$$

Note that only the last two terms are relevant when maximising expression (2). Therefore, solving the game played in the APA treatment is formally equivalent to solving a standard all-pay auction (i.e. without public good), in which bidders endowed with either 120, 160, 200 or 240 units compete for a prize of 240 units, and where budgets are private information. In order to solve such a game, it is helpful to first establish the following result.[5]

Lemma 1. *There are no Bayesian Nash equilibria in pure strategies.*

Proof. See Appendix B.

We therefore focus on mixed strategy equilibria. Although we cannot exclude there being multiple equilibria, we can restrict our attention to specific classes of equilibria. In particular, we can rule out the existence of symmetric equilibria, in which all players randomise according to the same distribution function independently of their endowment.

Lemma 2. *There exist no symmetric equilibria.*

Proof. See Appendix B.

In light of the above result, we concentrate on quasi-symmetric equilibria.[6] More specifically, we characterise an equilibrium in which different types randomise over different supports and expected bids are positively related to endowments. We refer to players with endowment 120, 160, 200 and 240 as w, x, y and z, respectively.

Proposition 2. *There exists a quasi-symmetric Bayesian Nash equilibrium in which*:

- *players of type w choose their contribution according to the distribution function $W(g) = (4g/15)^{\frac{1}{3}}$ in the interval $[0, 15/4]$;*

[5] All results are based on the assumption that ties are randomly broken, as in the experiment.

[6] We define an equilibrium as quasi-symmetric if players with the same endowment randomise according to the same distribution function.

Table 1

Theoretical Predictions: Absolute and Relative Contributions

	Incomes				Average
Treatments	120	160	200	240	180
			Absolute contributions		
VCM	0	0	0	0	0
LOT	45	45	45	45	45
APA	1	14	61	164	60
			Relative contributions		
VCM	0	0	0	0	0
LOT	38	28	23	19	25
APA	1	9	30	68	33

Note. Contributions are rounded to the nearest integer. Relative contributions are expressed as a percentage of the endowment.

- *players of type x choose their contribution according to the distribution function* $X(g) = (4g/15)^{\frac{1}{3}} - 1$ *in the interval* [15/4, 30];
- *players of type y choose their contribution according to the distribution function* $Y(g) = (4g/15)^{\frac{1}{3}} - 2$ *in the interval* [30, 405/4];
- *players of type z choose their contribution according to the distribution function* $Z(g) = (4g/15)^{\frac{1}{3}} - 3$ *in the interval* [405/4, 240].

Total expected group contribution is equal to 240 tokens.

Proof. See Appendix B.

From Proposition 2 we can calculate the expected value of each income type's contribution to the public good. In equilibrium, the expected contributions of players *w*, *x*, *y* and *z* are 15/16, 225/16, 975/16 and 2625/16 tokens, respectively.

Table 1 summarises the predicted contributions for the three treatments, both in absolute and relative terms, for each income type and on average.[7] First, average contributions for both prize-based mechanisms are higher than the predicted contribution in VCM, which is zero. They are also higher than the average provision in VCM, where an amount equivalent to the cost of the prize is used to finance the public good directly, resulting in an average provision of 30 tokens per subject. Second, the expected average group contribution in APA (60 tokens) is higher than in LOT (45 tokens). Third, the predicted absolute contributions are independent of income levels in LOT, whereas they rise steeply with income in APA, both in absolute and relative terms.

Summing up, the main hypotheses to be tested are as follows:

Hypothesis 1 – Absolute Efficiency. Both LOT and APA outperform VCM, not only in terms of gross contributions, but also after taking into account the cost of the prize.

[7] In all Tables and Figures, we will refer to expected contributions when reporting APA predictions.

Hypothesis 2 – Relative Efficiency. Total contribution to the public good is higher in APA than in LOT.

Hypothesis 3 – Individual income types. Individual contributions do not depend on income in LOT, whereas they rise with income in APA.

3. Results

This Section presents the experimental results. We start with a descriptive analysis of the main features of the data for the three treatments. Next, we examine the replicability of sessions within each treatment, the effects of repetition over rounds and the dependence of individual observations within sessions. We then present formal tests of the theoretical predictions, by comparing across treatments overall contribution to the public good and contributions by income types. Finally, we examine the performance of the three mechanisms at individual level.

3.1. *Overview*

Figure 1 displays average relative contributions, as a percentage of the endowment, over rounds for each treatment. Table 2 reports average relative contributions for each session over all 20 rounds and for selected sub-sets of rounds (1st, first 10, last 10 and 20th). The results for VCM sessions are similar to those generally obtained in public good experiments with homogeneous incomes. Average contributions to the group account are substantially higher than the equilibrium prediction of zero throughout the 20 rounds, but display a clear downward trend over successive rounds. Averaging over all sessions, individual contributions are 21.6% over the 20 rounds, falling from 35.1% in the first round to 8.2% in the last round. The same pattern of positive but declining contributions is observed in each of the three VCM sessions.

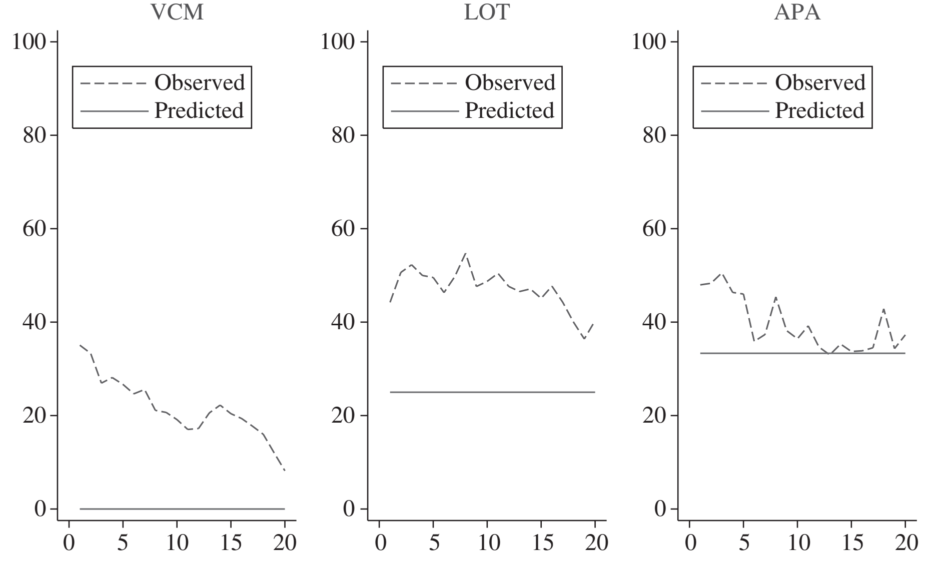

Fig. 1. *Average Relative Contributions Over Rounds, By Treatment*

Table 2

Average Individual Relative Contributions: By Session and Rounds

	Rounds				
Session	1–20	1	1–10	11–20	20
VCM 1	18.6	25.7	20.6	16.6	7.7
VCM 2	26.3	32.4	28.9	23.7	11.2
VCM 3	19.9	47.1	28.9	10.9	5.7
Average	21.6	35.1	26.1	17.1	8.2
LOT 1	42.1	38.5	45.3	38.9	39.9
LOT 2	46.2	48.5	50.0	42.3	39.3
LOT 3	52.7	45.7	52.8	52.5	41.8
Average	47.0	44.2	49.4	44.6	40.3
APA 1	41.8	51.8	46.0	37.7	52.4
APA 2	40.7	46.8	45.3	36.1	29.2
APA 3	36.2	45.3	38.5	33.9	30.3
Average	39.6	48.0	43.2	35.9	37.3

Note. Contributions to the public good are expressed as a percentage of the endowment.

Average contributions in LOT sessions are also systematically higher than the predicted contribution of 25% and remain virtually unchanged over successive rounds, except for a slight decline at the end of the sessions. Overall, the average contribution is 47%. In APA sessions, average contributions are higher than the predicted expected contribution of 33% and display a slight decline over successive rounds. The average overall contribution is 39.6% over the 20 rounds, falling from 43.2% in rounds 1–10 to 35.9% in rounds 11–20. All the APA sessions display a similar pattern of declining contributions that tend to converge to the theoretical prediction within the first ten rounds.

3.2. *Replicability, Repetition, Independence*

The descriptive analysis of session-level data indicates that contributions for individual sessions within each treatment are qualitatively similar in terms of both average levels and dynamics over rounds. Table 3 presents Kruskal-Wallis test statistics for the null

Table 3

Tests for Replicability of Sessions

	Rounds				
Treatment	1–20	1	1–10	11–20	20
VCM	4.93	5.83	4.21	6.43	0.75
	(0.08)	(0.05)	(0.12)	(0.04)	(0.69)
LOT	2.16	1.08	0.89	3.68	0.26
	(0.34)	(0.58)	(0.64)	(0.16)	(0.88)
APA	1.90	0.50	1.67	0.99	2.83
	(0.39)	(0.78)	(0.43)	(0.61)	(0.24)

Note. The Table reports Kruskal-Wallis test statistics for the null hypothesis that median contributions are equal across the three sessions within each treatment. p-values (in parenthesis) are based on the χ^2 distribution with 2 degrees of freedom.

hypothesis that median contributions are equal across the three sessions within each treatment (replicability), using the same sub-sets of rounds as in Table 2. Focusing on the whole session (rounds 1–20) or the last round, the results indicate that the null hypothesis cannot be rejected for all treatments. We therefore conclude that the three sessions can be pooled and the analysis is carried out on observations for 48 individuals for each of the three treatments.

The analysis of session-level data also indicates that there are substantial changes in contributions over successive rounds (repetition effects). Contributions tend to fall over rounds, generally converging towards theoretical predictions, in all treatments except for LOT, where the tendency to converge is less marked. Table 4 presents results of Wilcoxon signed-rank tests for the hypothesis that median contributions are the same across selected pairs of rounds within each treatment.

Irrespective of the time horizon considered, decreases in contributions are significant in VCM. LOT does not display any significant round effects, whereas in APA the differences are significant between rounds 1 and 10 (p-value 0.04) and marginally significant between rounds 1 and 20 (p-value 0.08). We conclude that repetition affects different mechanisms in different ways, so that comparisons between treatments, and between actual and predicted contributions within treatments, cannot focus on a single round but should consider alternative sub-sets of rounds in order to take into account the different role played by repetition in each treatment.

In order to go beyond descriptive analysis and provide formal tests of the theoretical predictions, we need to define the appropriate unit of analysis (subject, group, session). It is important to note that, because of repetition, subject-level observations within each session and round might be dependent, given that (in rounds beyond the first) subjects have interacted in previous rounds. In addition, because of the random rematching mechanism (at the beginning of each round subjects are randomly and anonymously rematched in groups of four people), independence could also be violated for group-level observations. If the dependence of subject-level observations due to interactions in earlier rounds were relevant, inference should be based on session-level observations (Orzen, 2008).

Table 4

Tests for Repetition Effects

	Rounds				
Treatment	1 vs 10	10 vs 20	1 vs 15	5 vs 20	1 vs 20
VCM	3.04 (0.00)	2.96 (0.00)	3.26 (0.00)	4.11 (0.00)	5.54 (0.00)
LOT	−1.42 (0.16)	1.65 (0.10)	−0.04 (0.97)	1.23 (0.22)	0.99 (0.32)
APA	2.04 (0.04)	0.16 (0.87)	1.95 (0.05)	−0.31 (0.76)	1.76 (0.08)

Note. The Table reports normalised Wilcoxon signed-rank test statistics for the hypothesis that median contributions are the same in the two rounds indicated in the column headings. p-values (in parenthesis) refer to two-sided tests based on the standard normal distribution.

However, the characteristics of the experimental design are such that the dependence across individual observations can be considered negligible. First, at the end of each round subjects only learn about the total contribution of other group members, so that it is difficult for them to infer individual absolute contributions. Second, since subjects do not know the endowments of other group members, it is even more difficult for them to infer other subjects' relative contributions (e.g. an absolute contribution of 120 could be a relative contribution of 50% as well as 100%, depending on the endowment of the other subject). Third, the number of subjects within each session (16) is such that each subject knows that there is a relatively small probability of interacting with the same subjects as in the previous round. This further reduces the motivation to reciprocate in successive rounds, thus weakening the possible dependence across individual observations.

We also investigated the issue at the empirical level, by considering Spearman rank correlation tests for the null hypothesis of independence between the contributions of each subject and the average contributions of the subjects who were in his group in the previous round. The test statistics, based on 16 individual observations for each session and each round, are significant at the 5% level in only about 15% of the cases. In addition, in the significant cases, 30% of the correlation coefficients are positive and 70% are negative, indicating that there is no systematic pattern in the relationship between each subject's contribution and those of his past group members. As a result, considering both the features of the experimental design and the results of the Spearman tests, we conclude that the dependence across individuals can be considered negligible. Hence, in the following, we use subjects as the unit of analysis; see Morgan and Sefton (2000) for a similar approach.

3.3. *Comparison Between Treatments: Total Contributions*

In order to compare the relative performance of the different funding mechanisms, it is necessary to consider contributions *net of the cost of prizes* in the prize-based mechanisms. Alternatively, it is necessary to take into account exogenous contribution in VCM (120 tokens per group, corresponding to 16.7 percentage points per subject in terms of relative contributions), thus focusing on overall public good *provision*. Figure 2 displays

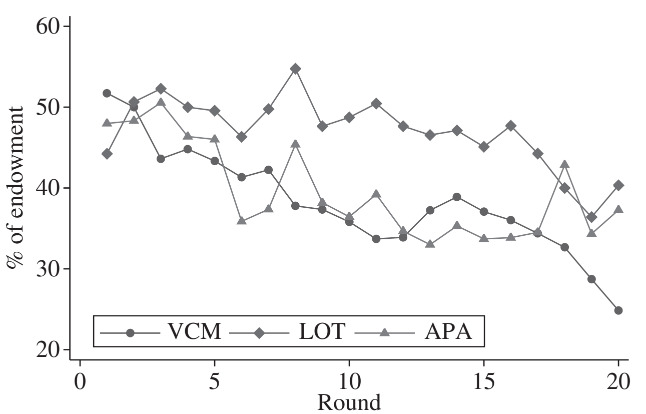

Fig. 2. *Average Public Good Provision Over Rounds*

Table 5

Tests of Equal Public Good Provision Between Treatments: All Subjects

Treatments	Rounds 1–20	1	1–10	11–20	20
LOT – VCM	4.55	−3.10	3.43	4.75	5.22
	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)
APA – VCM	1.88	−1.55	1.12	1.52	4.75
	(0.03)	(0.94)	(0.13)	(0.06)	(0.00)
APA – LOT	−4.09	1.52	−2.84	−4.39	−0.92
	(1.00)	(0.06)	(1.00)	(1.00)	(0.82)

Note. The Table reports Wilcoxon rank-sum tests (normalised z-statistics) for the hypothesis that the median of the difference between individual relative provision to the public good in the given two treatments is zero. p-values (in parenthesis), based on the standard normal distribution, refer to one-sided hypotheses as predicted by the theory.

average public good provision over rounds, providing the appropriate reference for comparing incentive-based mechanisms with the benchmark VCM.

While LOT systematically outperforms VCM (with the only exception of the first round), APA is very close to VCM, except for the last rounds, where the two profiles diverge. Averaging over all rounds, relative to VCM, public good provision is about 20% higher in LOT and 3.5% higher in APA.

The informal evidence presented in Figure 2 is examined further in Table 5, presenting results of Wilcoxon rank-sum tests of the null hypothesis that median public good provision is the same across treatments, under the assumption of independent individual-level observations. The first two rows compare each of the incentive-based mechanisms with the benchmark VCM. The third row compares APA with LOT. Given that our model predicts the direction of departure from the null hypothesis, we use the relevant one-sided hypotheses.

The test statistics are positive and strongly significant at all time horizons (except for round 1) in the comparison between LOT and VCM. APA also significantly outperforms VCM in terms of public good provision, although the significance level is quite variable across sub-samples, owing to the different effects of repetition in the two treatments.

Result 1. *Both the lottery and the all-pay auction are more effective than the voluntary contribution mechanism in funding public goods.*

The comparison between LOT and APA indicates that APA does not perform better than LOT, and indeed public good provision is significantly higher in LOT than in APA.

Result 2. *The lottery is more effective than the all-pay auction in funding public goods, contrary to the theoretical predictions.*

It is interesting to observe that incentive-based funding mechanisms are generally efficient in covering the cost of the prize. Averaging over all sessions and rounds for

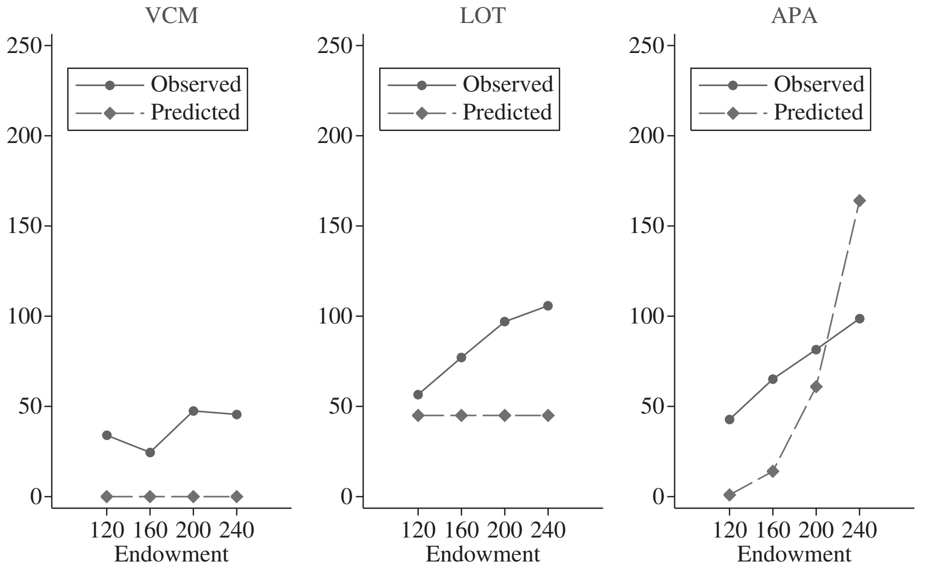

Fig. 3. *Average Absolute Contributions By Endowment*

each treatment, group contributions cover the cost of the prize in 96.3% of the cases in LOT and 88.3% of the cases in APA. This indicates that the lottery outperforms the all-pay auction also in terms of financial efficiency.

3.4. *Comparison Between Treatments by Income Level*

In this sub-section, we focus on the predictions for the contributions of individual income types (see Table 1). We first examine, within each treatment, whether individuals with different incomes behave according to the theoretical predictions. Next, we compare the performance of the three funding mechanisms for individual income levels.

Figure 3 displays average absolute contributions by income levels for each treatment. Over-contributions in VCM are observed for all income types, and rise slightly with income in absolute terms. In LOT, all income types over-contribute and, contrary to the theoretical predictions, absolute contributions rise almost linearly with income. In APA, absolute contributions rise with income, although not as steeply as predicted by the theory. As a consequence, the three lowest-income types over-contribute, while subjects with the highest income under-contribute.

Result 3. *Absolute contributions are positively related to income in both LOT and APA.*

Figure 4 provides a comparison of relative public good provision by income level in the three treatments.[8] Interestingly, the lottery outperforms both other mechanisms for all income types. Public good provision in APA is higher than in VCM for income types 160 and 240 but the opposite holds for the lowest income type. This indicates

[8] In order to ensure comparability across treatments, in VCM the 120 tokens of exogenous provision are attributed to each income-type on the basis of equal income shares (16.7%).

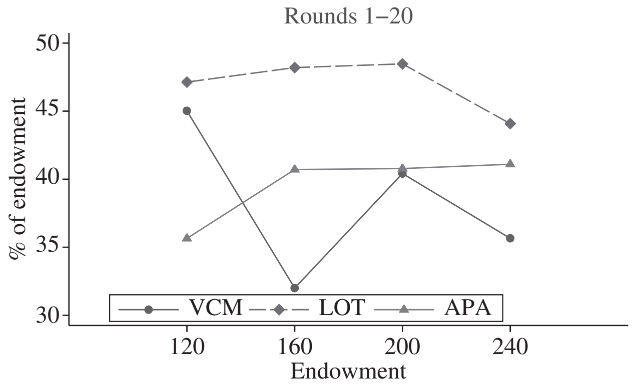

Fig. 4. *Average Relative Provisions by Endowment: All Treatments*

Table 6

Tests of Equal Public Good Provision Between Treatments, By Income Level

Treatments	Incomes 120	160	200	240
LOT – VCM	1.05	2.62	2.09	3.14
	(0.15)	(0.00)	(0.02)	(0.00)
APA – VCM	−1.05	2.62	0.00	1.32
	(0.85)	(0.00)	(0.50)	(0.09)
APA – LOT	−3.14	−1.57	−2.09	−0.52
	(0.00)	(0.06)	(0.98)	(0.70)

Note: The Table reports Wilcoxon rank-sum tests (normalised z-statistics) for the hypothesis that the median of the difference between individual contributions in the given two treatments is zero. p-values (in parenthesis), based on the standard normal distribution, refer to one-sided tests as predicted by the theory.

that, in a deterministic contest, a prize provides a relatively less effective incentive for poorer individuals.

Table 6 presents results of Wilcoxon rank-sum tests of the null hypothesis that median public good provision is the same across treatments, when considering separately each income type. The results, based on the assumption of independent individual-level observations, indicate that LOT performs significantly better than VCM for all income types, although only marginally for the lowest income type. LOT also performs significantly better than APA for incomes 120, 160 and 200. APA does significantly better than VCM only for income type 160.

Result 4. *The lottery outperforms the other funding mechanisms throughout the income distribution.*

3.5. *Comparison Between Treatments at Individual Level*

Finally, we consider the relative performance of the three mechanisms at individual level. Figure 5 compares the cumulative distribution functions of relative

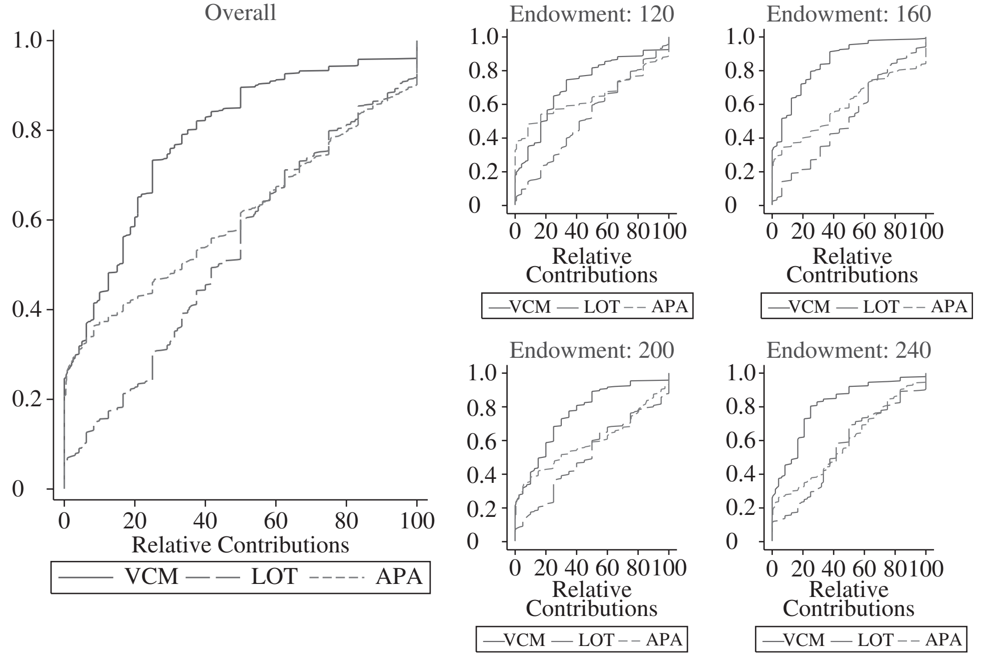

Fig. 5. *Distribution of Contributions By Treatment: Overall and By Endowment*

contributions for the three treatments, overall and by income type. The main difference between the two prize-based mechanisms is that subjects choose zero contributions about three times as often in APA as in LOT (20.8% and 5.83%, respectively). The cumulative distribution for APA lies above that for LOT only up to a relative contribution of 50%, while the two distributions are virtually identical thereafter.

Result 5. *At the individual level, APA is characterised by a much higher fraction of zero contributions than LOT.*

Focusing on individual income types, it is interesting to observe that the difference between APA and LOT in the frequency of low contributions is very pronounced for low income types but it becomes less and less evident for higher income types. In LOT subjects contribute almost uniformly irrespective of their income type. On the other hand, in APA low-income individuals contribute zero, or close to zero, much more often than high-income subjects. Nevertheless, subjects of all types randomise on all their support (from zero to their own endowment), differently from what is predicted in the equilibrium characterised in Proposition 2.

4. Discussion

Our experimental results indicate that, contrary to the theoretical predictions, contributions to the public good are higher in the lottery than in the all-pay auction, both overall and by individual income types. This finding may suggest a number of interpretations. For instance, since subjects are generally more familiar with lotteries than with all-pay auctions, they might tend to bid more conservatively in the latter. Alternatively, it could be argued that, in LOT, subjects may be willing to 'give chance a chance' and make positive bids even if they expect others to bid more aggressively. On the other hand, in an all-pay auction, subjects who expect to be outbidded would prefer to bid zero. These interpretations are supported by the finding that, at the individual level, subjects choose zero contributions in APA three times as often as in LOT.

In this Section, we focus on an interpretation of the results based on the effects of competition on agents' concern for social efficiency. We provide a framework that allows explanation of the discrepancies between the theoretical predictions and our experimental results. We also show that this framework can be used to explain some of the results of closely related experiments in the literature. Let us start by making two preliminary observations.

First, in the absence of the public good component, experimental data support the theoretical prediction that more rents are dissipated in all-pay auctions than in lotteries (Davis and Reilly, 1998). The opposite pattern is observed, instead, in Orzen (2008) and in our experiment, where agents contribute to a public good: bids are higher in a stochastic framework than in a perfectly discriminatory one.[9] This puzzling finding is due to over-bidding in the lottery, while the all-pay auction's results are on average compatible with the predictions in both experiments. Furthermore, over-contribution relative to equilibrium is larger in the VCM than in the lottery treatment. These results suggest that

(*a*) the discrepancy between data and predictions is related to the presence of the public good;
(*b*) over-contributions are smaller the more competitive the environment.

We also note that subjects with larger endowments make higher contributions both in the VCM and in the lottery treatments, contrary to the theoretical predictions.

The second observation relates to Morgan and Sefton (2000). Their study compares different lottery treatments, where it is always socially optimal to contribute everything. They observe that, when the equilibrium is suboptimal but close to efficiency, the data conform to the prediction, while when the equilibrium is well below efficiency, subjects significantly overbid. As they point out: 'It is intriguing that other-regarding preferences may substitute for efficient incentives; if so, theoretical comparisons may overstate the efficiency gains of the lottery mechanism' (Morgan and Sefton, 2000, p. 798). It is suggestive that these results conform to recent experimental findings showing that subjects are concerned with increasing social welfare, even at the cost of sacrificing their own payoff (Charness and Rabin, 2002).

On the basis of these two observations, we propose a simple framework that takes the link between the degree of competition and agents' concern for social efficiency explicitly into account. This framework allows us to provide an interpretation of the

[9] More precisely, we find a statistically significant difference between LOT and APA, whereas Orzen (2008) reports higher bids in the lottery but the difference is not significant.

experimental results. Consider a linear public good game, with heterogenous endowments, where it is socially efficient to contribute the entire endowment. A prize is either awarded through a contest (stochastic or deterministic), or shared equally by all the players. Suppose that each player i maximises the following utility function

$$V_i = \pi_i - \frac{\alpha}{\rho + 1} \frac{(\omega_i - g_i)^2}{\omega_i} \tag{3}$$

where π_i represents i's material payoff, either in a VCM, a lottery or an all-pay auction. ω_i and g_i represent i's endowment and contribution, respectively, while α is a finite non-negative parameter. ρ is the exponent of a Tullock's contest success function (Tullock, 1980). It is therefore equal to 1 in the case of a lottery, it approaches infinity in an all-pay auction, while it is equal to 0 in the VCM, where the prize is shared equally by all players independently of their contributions.[10]

The term $\alpha(\omega_i - g_i)^2/\omega_i$, based on our second observation, captures the psychological cost of contributing less than what is socially optimal. The further an individual's contribution from the social optimum, the higher the cost. We assume that, for a given contribution and a given α, the cost is higher for an individual with higher endowment. The term $1/(\rho + 1)$, based on our first observation, is the weight of the psychological cost. We assume that competition has a negative effect on the concern for social efficiency in the utility function. More precisely, the weight of the psychological cost of contributing less than what is socially optimal decreases with the degree of competition (captured by ρ): it is equal to 1 in the VCM case, where there is no competition, $\frac{1}{2}$ in the lottery, where there is an intermediate degree of competition, while it falls to zero in the all-pay auction, where competition is strongest.

The description of preferences outlined above can be used to interpret our experimental results. Consider first the VCM treatment. An agent who maximises his own material payoff would not contribute to the public good. However, maximising (3) with respect to g_i, we obtain

$$g_i^* = \frac{2\alpha - 1}{2\alpha} \omega_i.$$

The average contribution for the VCM treatment in our experiment is around 22% of the endowment. Setting $g_i/\omega_i = 0.22$ we obtain an estimated α of 0.64.

Let us now turn the attention to the lottery game, where $\rho = 1$. Call the players with endowment 120, 160, 200 and 240, w, x, y and z respectively, while g_w, g_x, g_y and g_z denote their contributions.

PROPOSITION 3. *When* $\alpha = 0.64$, *the lottery has a unique pure strategy Bayesian Nash equilibrium such that* $g_w^* = 61$, $g_x^* = 71$, $g_y^* = 78$ *and* $g_z^* = 85$.

Proof. See Appendix B.

[10] If agents are risk-neutral, this is in fact equivalent to the case in which the prize is randomly assigned, independently of the players' contributions. Hence, $\rho = 0$ indicates a contest with the highest possible noise.

Notice that these predictions fit the experimental data much better than those presented in Section 3. First, they are in line with the observation that contributions increase with the endowment. Second, because the psychological cost embedded in (3) goes to zero as ρ approaches infinity, the predictions for the all-pay auction are unchanged. Therefore, the model presented above predicts a higher total expected contribution in the lottery (295 tokens) than in the all-pay auction (240 tokens), consistent with our experimental results.

We now wish to check whether this model can help to explain the puzzles raised by some of the results in Morgan and Sefton (2000) and Orzen (2008). We start by considering three treatments presented in Morgan and Sefton (2000), called I-LOT, P-LOT and BIGLOT. The Nash equilibrium predicts that agents would contribute 8 tokens out of 10 in P-LOT and 12 out of 20 in BIGLOT. This is relatively close to efficiency and the predictions fit the data quite well (slightly overestimating the results in P-LOT and underestimating them in BIGLOT). However, in I-LOT the Nash equilibrium predicts that subjects will contribute 6 tokens out of 20, while they actually wager around 11 tokens. Setting $\alpha = 0.64$, our model predicts a contribution of around 8.5 tokens in P-LOT, 15.5 in BIGLOT and 13 tokens in I-LOT. These predictions remain quite close to the experimental observations for the cases of P-LOT and BIGLOT. Moreover, the model also predicts the subjects' behaviour in the I-LOT treatment better than the standard Nash equilibrium.

Next, consider the experimental results in Orzen (2008). In the lottery treatment, agents contribute around 40% averaging over all 20 rounds, and 33% considering the last 15 rounds. While in the standard Nash equilibrium agents should wager 18.75% of their endowment, our model (for $\alpha = 0.64$) predicts an average contribution of exactly 33%. This is a much better prediction of the subjects' observed contributions.

Summing up, the simple model presented above can help to interpret not only the discrepancies between our own experimental results and the theoretical predictions but also some previous unexplained results in the literature.

5. Concluding Remarks

This article presents an experimental investigation of the performance of prize-based public good funding mechanisms, when subjects have heterogenous endowments which are private information. We compare a lottery and an all-pay auction, while also considering a voluntary contribution mechanism as a benchmark. The results are only partially consistent with the theoretical predictions.

On the one hand, as predicted by the theory, both the lottery and the all-pay auction outperform voluntary contribution after taking into account the cost of the prize. This is an important result: in a setting where agents have heterogeneous incomes that are private information, prize-based fund-raising mechanisms can provide an effective way of overcoming free riding.

On the other hand, the comparison between the prize-based mechanisms indicates that, contrary to the theoretical predictions, contributions to the public good are significantly higher in the lottery than in the all-pay auction, both overall and by individual income types. This result may suggest a number of possible interpretations. Nevertheless, the interesting point is that, focusing on total contributions to the public good, while the all-pay auction does better than the theoretical prediction over all rounds, contributions are very close to the prediction in the last 10 rounds. Theory is contradicted mainly by over-contributions in the lottery.

Focusing on income heterogeneity, over-contributions are observed for all income types in VCM and are slightly increasing with income in absolute terms. In the all-pay auction, absolute contributions rise with income, even though not as steeply as predicted by the quasi-symmetric Nash equilibrium we characterised. In the lottery, all income types over-contribute and, contrary to the theoretical predictions, absolute contributions rise linearly with income. The latter result indicates that, from a theoretical perspective, the completely symmetric equilibrium of a lottery game does not seem to describe the actual behaviour of subjects properly.

We offered an interpretation of the discrepancies between theoretical predictions and experimental results based on the hypothesis that competition has a negative effect on agents' concern for social efficiency. We showed that a simple behavioural model that takes into account this effect can explain, in particular, why more competitive mechanisms, such as an-all-pay auction, tend to be outperformed by relatively less competitive mechanisms, such as a lottery. The model is also able to account for the positive relationship between endowments and contributions in both VCM and the lottery. Finally, we showed that our framework can help to interpret some of the unexplained findings of closely related experiments in the literature.

Appendix

A. *Instructions*

[ALL TREATMENTS]

Welcome. Thanks for participating in this experiment. If you follow the instructions carefully and make good decisions you can earn an amount of money that will be paid to you in cash at the end of the experiment. During the experiment you are not allowed to talk or communicate in any way with other participants. If you have any questions raise your hand and one of the assistants will come to you to answer them. The rules that you are reading are the same for all participants.

General rules

There are 16 people participating in this experiment. At the beginning of the experiment each participant will be assigned randomly and anonymously an endowment of either 120, 160, 200, or 240 tokens with equal probabilities.

The experiment will consist of 20 rounds. In each round you will have the same endowment that has been assigned to you at the beginning of the experiment. In each round you will be assigned randomly and anonymously to a group of four people. Therefore, of the other three people in your group you will not know the identity and the endowment, that could be 120, 160, 200, or 240 tokens with equal probabilities.

How your earnings are determined

In each round you have to decide how to allocate your endowment between an INDIVIDUAL ACCOUNT and a GROUP ACCOUNT, considering the following information:

- for each token that you allocate to the INDIVIDUAL ACCOUNT you will receive 2 points.
- for each token allocated to the GROUP ACCOUNT (by you or by any other of the members of your group), every group member will receive 1 point.

[VCM]

In each round you will receive 120 bonus points.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted into euro at the rate 1000 points = 1 euro. The resulting amount will be paid to you in cash.

[LOT]

In each round you can win a prize of *240 points* on the basis of the following rules. For each token allocated to the GROUP ACCOUNT you will receive a lottery ticket. At the end of each round the computer selects randomly the winning ticket among all the tickets purchased by the members of your group. The owner of the winning ticket wins the prize of 240 points. Thus, your probability of winning is given by the number of tokens you place in the GROUP ACCOUNT divided by the total number of tokens placed in the GROUP ACCOUNT by members of your group. In the case where no tokens are placed in the GROUP ACCOUNT, the winner of the prize is selected randomly among the four members of the group.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in euro at the rate 1000 points = 1 euro. The resulting amount will be paid to you in cash.

[APA]

In each round you can win a prize of *240 points* on the basis of the following rules. The member of your group who allocates the highest amount to the GROUP ACCOUNT is the winner of the prize. In the case of ties among one or more group members, the winner is determined randomly.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted into euro at the rate 1000 points = 1 euro. The resulting amount will be paid to you in cash.

B. *Proofs*

Proof of Proposition 1. If player i with endowment ω_i contributes $g_i \in [0, \omega_i]$ he wins 240 with probability $g_i/(g_i + G_{-i})$, where G_{-i} is the sum of the contributions of all the other agents. Player i's expected utility is given by

$$2\omega_i + G_{-i} + 240\frac{g_i}{g_i + G_{-i}} - g_i.$$

Differentiating with respect to g_i and setting equal to zero we obtain

$$240\frac{G_{-i}}{(g_i+G_{-i})^2}-1=0.$$

Assuming that total contribution is different from zero[11] and rearranging we obtain player i's best response function, given by the following expression

$$g_i^* = -G_{-i} + \sqrt[2]{240G_{-i}}. \quad (4)$$

Based on (4) we can write an expression for the total contribution when player i plays according to his best response function

$$G(g_i^* \mid G_{-i}) = \sqrt[2]{240G_{-i}}.$$

Although endowments are private information, notice that ω does not enter the first order condition. Each player will have the same best response function and the contribution in equilibrium will be independent of ω. Therefore we know that

$$g_i^* = \frac{\sqrt[2]{240G_{-i}}}{4}. \quad (5)$$

Setting (4) and (5) equal, we obtain that $G_{-i} = 135$, when all players play according to the best response functions. Therefore, in equilibrium all agents contribute $g^{LOT} = 45$ tokens and total contribution is $G^{LOT} = 180$ tokens.

Proof of Lemma 1. Suppose that there exists a pure strategy Bayesian Nash equilibrium such that player i, with the highest possible endowment, bids g_i and g_l,g_{h-1} and g_h represent, respectively, the lowest bid, the second highest bid and the highest bid among i's possible opponents. There can be two cases. First, suppose that $g_l = g_h$. Given that i faces opponents with the lowest possible endowment with positive probability, g_l must be on the interval [0,120], meaning that i can always outbid player l. In this case i could profitably deviate to $g_l + \varepsilon$. On the other hand, suppose that $g_l \leq g_{h-1} < g_h$. If $g_h < g_i$ or if $g_h = g_i < 240$ then i could deviate and play $g_h + \varepsilon$, with ε arbitrarily small, increasing his expected payoff. If $g_h > g_i$ or if $g_h = g_i = 240$ then i would be better off playing $g_{h-1} + \varepsilon$. This rules out the existence of pure strategy Bayesian Nash equilibria.

Proof of Lemma 2. In order to prove Lemma 2, we first need to present the following auxiliary lemma.

LEMMA 3. *In equilibrium no player can have a mass point at any bid* $g \in (0, 240)$.

Proof. Suppose, to the contrary, that exactly one player has a mass point at $g \in (0, 240)$. There is an interval $(g - \varepsilon, g)$, with ε arbitrarily small, where no other agent puts positive density. If another player put density in that interval, moving all the density from $(g - \varepsilon, g)$ to g would guarantee a discrete increase in the probability of winning at an infinitesimal cost. However, if no one else puts density on $(g - \varepsilon, g)$, then the bidder with mass at g would have an incentive to move his mass lower. Now suppose that more than one player has a mass point at $g \in (0, 240)$. Then there would be at least one player who could move his mass at $g + \varepsilon$, increasing his expected payoff.

[11] Notice that in equilibrium total contribution will not be zero. In fact, if any other player different from i contributes zero, player i will contribute ε arbitrarily close to zero and win the prize.

Let us return to the proof of Lemma 2. Suppose, contrary to the hypothesis, the existence of a symmetric equilibrium. Then, all players must share the same supremum $\bar{g} \leq 120$. Since Lemma 0 shows that there cannot be mass points at $\bar{g}$, it follows that, when bidding $\bar{g}$, each player earns an expected payoff of at least 120. This means that the bidders must put a positive mass at their common infimum, otherwise their expected surplus could not be strictly positive. Therefore, we know from Lemma 0 that the players' infimum must be equal to zero. Suppose that the players bid zero with probability p. When bidding zero, each player earns an expected surplus equal to $p^3(240/4)$, which is strictly less than the expected payoff from bidding $\bar{g}$. As in equilibrium each player must be indifferent among all bids to which he assigns positive probability, this contradicts our initial assumption.

Proof of Proposition 2. Call $W(g)$, $X(g)$, $Y(g)$ and $Z(g)$ the distribution functions according to which players of type w,x,y and z, respectively, choose their contributions. Suppose that players of type w, x, y and z randomise over the supports $[a, b]$, $[c, d]$, $[e, f]$ and $[h, i]$, respectively, with $0 \leq a < b \leq c < d \leq e < f \leq h < i \leq 240$. Moreover, suppose that, in equilibrium, no agent earns a strictly positive expected surplus.

Consider player j of type z. Recalling that j's opponents can be of type w, x, y or z with equal probability, j faces three players with lower endowments with probability 27/64; two opponents with lower budgets and one with his same endowment with probability 27/64; one player with lower budget and two with his same endowment with probability 9/64; finally, j faces three opponents with his same budget with probability 1/64. Hence, j's payoff from bidding $g \in [h, i]$ will be

$$240\left\{\frac{27}{64} + \frac{27}{64}Z(g) + \frac{9}{64}[Z(g)]^2 + \frac{1}{64}[Z(g)]^3\right\} - g.$$

In a mixed strategy equilibrium j must be indifferent among all bids to which he assigns positive probability. As we assumed full dissipation of rents, this implies that the following equation must hold

$$240\left\{\frac{27}{64} + \frac{27}{64}Z(g) + \frac{9}{64}[Z(g)]^2 + \frac{1}{64}[Z(g)]^3\right\} = g.$$

The equation above can be rewritten as

$$\frac{15}{4}[3 + Z(g)]^3 = g$$

which means that z players randomise according to $Z(g) = (4g/15)^{\frac{1}{3}} - 3$ on the interval [405/4, 240].

Consider now player j of type y. Proceeding as we have done for type z we can calculate j's payoff from playing $g \in [e, f]$. Hence, in equilibrium the following must hold

$$240\left\{\frac{8}{64} + \frac{12}{64}Y(g) + \frac{6}{64}[Y(g)]^2 + \frac{1}{64}[Y(g)]^3\right\} = g.$$

This means that type y will randomise according to $Y(g) = (4g/15)^{\frac{1}{3}} - 2$ on the interval [30, 405/4]. Notice that f, y's supremum, coincides with h, z's infimum.

Considering player j of type x, when j bids $g \in [c, d]$ the following equation must hold

$$240\left\{\frac{1}{64}+\frac{3}{64}X(g)+\frac{3}{64}[X(g)]^2+\frac{1}{64}[X(g)]^3\right\}=g.$$

Therefore j will choose his contribution from the distribution function $X(g) = (4g/15)^{\frac{1}{3}} - 1$ on the interval [15/4, 30]. Note that d coincides with e.

Finally, consider player j of type w. This player can only win the prize when facing three opponents of his own type, which happens with probability 1/64. Hence, when he plays $g \in [a, b]$ it must be the case that

$$240\left\{\frac{1}{64}[W(g)]^3\right\}=g.$$

Therefore type w randomises according to $W(g) = (4g/15)^{\frac{1}{3}}$ on the interval [0,15/4]. Note that b is equal to c and that $a = 0$.

By construction, if all other agents choose their bids according to the distributions described above, player j of type $t \in \{w, x, y, z\}$ has no incentive to deviate and bid $\hat{g} \in [0, t]$. As rents are completely dissipated, total expected revenue is equal to 240.

Proof of Proposition 3. Consider the generic type t, with endowment ω_t, who contributes g_t. Type t faces different combinations of opponents with given probabilities, each combination representing a state of the world. The generic state of the world $s \in \{1,\ldots,S\}$ is identified by a triple (s_1, s_2, s_3) such that $s_i \in \{w, x, y, z\} \forall i \in \{1, 2, 3\}$. Call $\mathbf{g}^s$ the vector (g_1^s, g_2^s, g_3^s) of contributions associated to s such that $g_i^s \in \{g_w, g_x, g_y, g_z\} \forall i \in \{1, 2, 3\}$. Let $p^s > 0$ denote the probability associated to s, with $\sum_{s=1}^{S} p^s = 1$. Type t maximises the following expected utility function

$$2(\omega_t - g_t) + 240\sum_{s=1}^{S} p^s \frac{g_t}{g_t + \sum_{i=1}^{3} g_i^s} + g_t + \sum_{s=1}^{S} p^s \sum_{i=1}^{3} g_i^s - \frac{\alpha}{2}\frac{(g_t - \omega_t)^2}{\omega_t}. \tag{6}$$

Given the setting of our experiment (i.e. each player has equal probability of being either of type w, x, y or z) we calculate S, the number of possible states of the world, and the probability p^s associated to each event. We differentiate (6) with respect to g_t and set it equal to zero. Substituting g_t and ω_t with g_w and 120, g_x and 160, g_y and 200, and g_z and 240, respectively, we obtain four first order conditions, one for each type. When $\alpha = 0.64$ the system has a unique real root, representing the unique pure strategy Bayesian Nash equilibrium of the game, in which $g_w^* = 61$, $g_x^* = 71$, $g_y^* = 78$ and $g_z^* = 85$. Total expected contribution is equal to 295.

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